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Customer selection rules in competitive facility location

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Abstract. In a competitive facility location setting, customers have a wide range of options to choose from when deciding which facility to patronize. This decision-making process is influenced by a variety of factors, including price, location, quality of service, reputation, and other amenities.

It is important for businesses to understand the different selection rules that customers use and to carefully select the best approximation to build models when decisions need to be made. It is also important for developing effective marketing and branding strategies. By addressing the needs and preferences of their target audience, companies can increase their chances of attracting and retaining customers, and ultimately gain a competitive advantage in the marketplace.

This study discusses the most important customer selection rules and introduces a general modelling scheme for them. A new hybrid rule is proposed to include many patronising behaviours.

Keywords: competitive location, customer choice rules

1. Introduction

When locating a new facility, one of the most important considerations is whether there are competitors in the market offering the same goods or services. If there are competitors in the area, then the locating firm will have to compete for the market, and the profit that the firm makes will be affected by the decisions of its competitors. Therefore, maximizing profit is a much more difficult problem to solve in the presence of competitors than in a monopolistic scenario.

Knowing how customers split their purchases between existing facilities helps to estimate the market share captured by each facility (see [3–5]). The existing customer selection rules assume that all customers follow the same selection rule,

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however it is clear that in reality, it is not the case. There could be customers for each selection rule, and those should be taken into account in a competitive facility location problem. The aim of the present paper is thus to build a model where multiple customer selections rules are considered, and the new facility is sought accordingly.

If a chain is planning to locate a new facility, it is also important to know the type of competition it faces. If the characteristics of the competitors are known in advance and assumed to be fixed, static competition is assumed. However, the chain may anticipate that competitors will react by also locating a new facility, leading to Stackelberg-type models. This case is considered as competition with foresight, where the location of the leader is optimized, assuming that the follower also locates optimally. Considering dynamic competition assumes that there is an action-reaction cycle of the competing firms. In such settings, decisions are very difficult and can only be approached from a game theory perspective, thus strategies and equilibrium are sought.

This work deals with static competitive facility location problems, where demand is assumed to be inelastic and concentrated in a finite set of demand points. The attraction function considered is multiform, i.e. the facilities differ in location as well as in other aspects such as floor area, number of counters, parking, product mix, etc. The quality of the facility $j \in J$ as perceived by customer $i \in I$ follows the Multiplicative Competitive Interaction pattern [2] and is thus defined by

$$A_{ij} = \prod_k f_{ijk}^{\alpha_k},$$

where the kth accounted factor is measured in f_{ijk} and has importance α_k.

Now the attraction follows the Huff rule, depending on both the location and the quality of the facilities, being inversely proportional to a modified distance measure and proportional to the quality or other positive factors taken into account. In fact, the attraction (or utility) of facility j for customer i is expressed as

$$u_{ij} = \frac{A_{ij}}{g(d_{ij})},$$

where d_{ij} is the distance between facility j and customer i and $g(\cdot)$ is a non-negative, non-decreasing function that modifies the distance. Prices are not considered as decision variables, but they can be considered as part of the attraction factors that determine the qualities of the facilities. Most of the existing models of this type focus on market share maximisation [13–15], although profit maximisation has also been used in many recent works [7, 8].

A number of different customer selection rules have been presented in the literature and are now reviewed in the next section.

2. Patronizing behaviours

Patronizing behaviour is the way customers choose which facilities to favour based on their utility. First, we review the most relevant choice rules presented in literature, together with their models, using the notations introduced previously.

2.1. Binary or deterministic choice rule

An often used rule is that customers only travel to the nearest/cheapest facility to make their purchases, as occurs in Hotelling-like models [13]. It was the first rule introduced, and since equivalent products were assumed, only distance played a role. However, this role can also be based on the utility of the facilities, so that the most attractive facility gets all the demand. The market share of facility j can be calculated as

$$ms_j = \sum_{i \in I : u_{ij} > \max_{k \in J} u_{ik}} w_i$$

where w_i is the demand of customer i.

In the case of a tie, a tie-breaking rule is considered, which can be *New oriented*, where the new facility takes all the demand; *Conservative*, where the old facility takes all the demand; and a *Tie rule* can also be considered, where the demand is split between all the tied facilities according to the given rule.

2.2. Probabilistic rule

Another very frequently used rule in retailing is that each customer patronizes all available facilities offering the goods probabilistically, with a probability proportional to her/his attraction to each facility, as in Huff-like models [14, 15].

Using the probabilistic rule, the market share of facility j is

$$ms_j = \sum_{i \in I} w_i \frac{u_{ij}}{\sum\limits_{k \in J} u_{ik}}.$$

The probabilistic rule is used for instance in [7, 11].

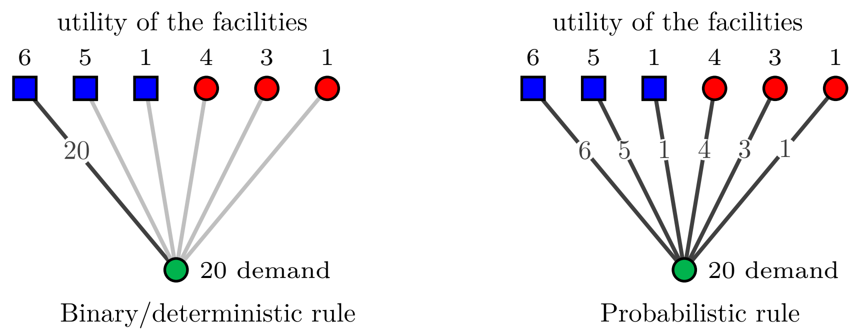

Figure 1. Example for the Binary and Probabilistic selection rules.

As an example of the two classic rules, see Figure 1, where there are two competing chains, denoted by squares and circles, with 3-3 facilities. Our customer's demand is 20, and the utility of the facilities for this customer is written above each facility. The demand that each facility gains is shown on the edge between them (0 if nothing is written).

2.3. Multi-deterministic or Partially binary rule

This rule is important when there are multiple chains in the market [6]. For a general setting, we can assume that L is the set of chains, and each chain l has its set of facilities C_l. We assume that the set of all facilities $J = \bigcup_{l \in L} C_l$.

The customer splits his demand between all chains, but is served by only the most attractive facility from each chain. The demand is shared probabilistically among the most preferred facilities of each firm. For chain l, its total market share is given by

$$ms_l = \sum_{i \in I} w_i \frac{\max\limits_{j \in C_l} u_{ij}}{\sum\limits_{k \in L} \max\limits_{j \in C_k} u_{ij}}.$$

2.4. Partially probabilistic rule

Using the probabilistic rule, all facilities are patronized, even those with very low utility. This is not realistic, as customers tend to split their demand between facilities with high utility. Therefore, the partially probabilistic rule [9] aims to solve this issue: only the facilities with a minimum level of utility $\underline{u}$ will serve the customer, and the facilities that do not reach the minimum utility are left without demand. Among the facilities with higher utility, the demand is split probabilistically. Thus, the market share for facility j can be written as

$$ms_j = \sum_{i \in I} w_i \frac{U_{ij}}{\sum\limits_{k \in J} U_{ik}} \quad \text{where} \quad U_{ij} = \begin{cases} u_{ij}, & \text{if } u_{ij} > \underline{u}, \\ 0, & \text{otherwise.} \end{cases}$$

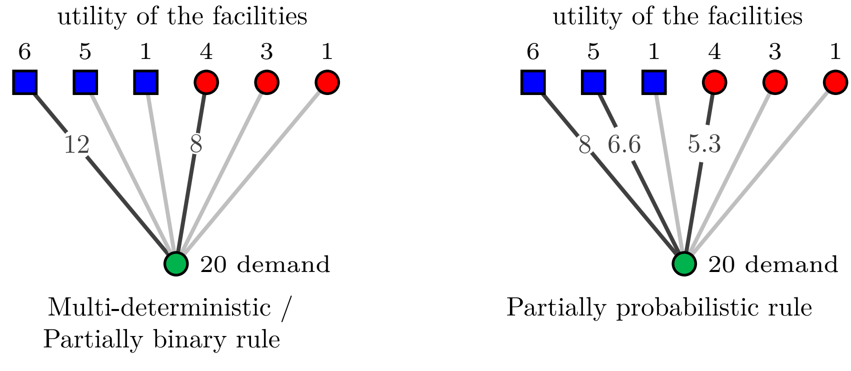

Figure 2. Example for the Multi-deterministic and Partially probabilistic selection rules.

An example of the Multi-deterministic and Partial probabilistic rule is shown in the graphs in Figure 2, where, as before, there are two competing chains, denoted by squares and circles, with 3-3 facilities. Our customer's demand is 20, and the utility of the facilities for this customer is written above each facility. The demand of each facility is written on the edge between them.

2.5. Pareto-Huff selection rule

This rule is based on the assumption that quality cannot be compared with distance. However, those facilities that are dominated by another facility, that is closer and of higher quality, should not be patronized by the customer. Thus, by selecting only the Pareto optimal facilities (minimizing distance and maximizing quality), the dominated facilities can be disregarded. Only the Pareto-optimal facilities, collected in the set P_i, can serve customer i. Among these facilities $j \in P_i \subseteq J$ for customer i, the demand is split probabilistically [10].

The market share of facility j is then

$$ms_j = \begin{cases} \sum\limits_{i \in I} w_i \dfrac{u_{ij}}{\sum\limits_{k \in P_i} u_{ik}} & \text{if } j \in P_i, \\ 0 & \text{otherwise.} \end{cases}$$

2.6. Brand preference

For some products, customers tend to choose by brand rather than by other factors. Therefore, a customer i splits his demand probabilistically among all facilities of his favourite brand $B(i)$. For a facility $j \in C_{B(i)}$, its market share is defined as

$$ms_j = \sum_{i \in I} w_i \frac{u_{ij}}{\sum\limits_{k \in C_{B(i)}} u_{ik}}.$$

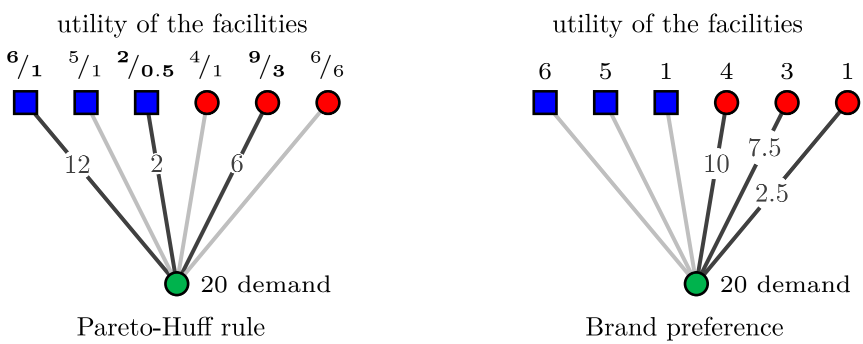

Figure 3. Example for the Pareto-Huff and Brand preference selection rules.

An example of the Pareto-Huff and Brand preference rules is shown in Figure 3, where the settings are the same as in Figures 1–2, except that for the Pareto-Huff case we have written the utilities as quality / distance, and highlighted in bold those in the Pareto-optimal front. For the Brand preference example, our client prefers the red circle chain to the blue square and divides his demand according to the utilities.

2.7. Covering-based choice rule

For some customers, or in some applications, distance is an important consideration and more distant facilities may not be accessible to customers. Thus, whenever a coverage-based customer selection rule is used, only those facilities within a given radius R are probabilistically patronized.

Therefore, the market share of facility j can be formalized as

$$ms_j = \sum_{i \in I} w_i \frac{U_{ij}}{\sum\limits_{k \in J} U_{ik}} \quad \text{where} \quad U_{ij} = \begin{cases} u_{ij}, & \text{if } d_{ij} \le R, \\ 0, & \text{otherwise,} \end{cases}$$

where it is assumed that all customers have at least one facility within the radius R, otherwise a dummy facility should take all their demand.

2.8. Multinomial Logit model (MNL)

It is a model based on random utility, where utility depends on some measurable characteristics, v_{ij}, but also on some random features, ε_{ij}, so $U_{ij} = v_{ij} + \varepsilon_{ij}$.

If we assume that ε_{ij} are identically independently distributed with the log-Weibull (also known as Gumbel) distribution, which allows us to express the probabilities of customer i to select a facility j as

$$\text{prob}_{ij} = \frac{e^{v_{ij}}}{\sum\limits_{k \in J} e^{v_{ik}}}.$$

This means that in this case we can set the utility as $U_{ij} = e^{v_{ij}}$, and so again, the market share of a facility j looks almost equivalent to the probabilistic rule, i.e.

$$ms_j = \sum_{i \in I} w_i \frac{U_{ij}}{\sum\limits_{k \in J} U_{ik}}.$$

The MNL rule is well studied in many papers, see [12] for some linearization approaches.

The example of the last rule can be the same as for the probabilistic rule in Figure 1, where we assume that the utilities are directly the $e^{v_{ij}}$ values.

Our first goal is the unification of the above rules, if possible, and the construction of a possible solution procedure on the basis of the unified model.

3. Unification of customer selection rules

Looking at the formulas for the different rules, we noticed that most of them are similar to the formula for the probabilistic rule. Therefore, we chose to generalize it to fit each rule. Let us denote by U_{ijr} the utility of a customer i for a facility j and a customer selection rule r, i.e.

$$U_{ijr} = \begin{cases} u_{ij}, & \text{if condition set by rule } r \text{ holds for } u_{ij}, \\ 0, & \text{otherwise.} \end{cases}$$

We can now write the market share of facility j for a given rule r

$$ms_{jr} = \sum_{i \in I} w_i \frac{U_{ijr}}{\sum\limits_{k \in J} U_{ikr}}.$$

There is another way to do that, namely

$$ms_{jr} = \sum_{i \in I_{jr}} w_i \frac{u_{ij}}{\sum\limits_{k \in S_{ir}} u_{ik}}, \tag{3.1}$$

where

$$S_{ir} = \{j \in J \mid \text{condition set by rule } r \text{ holds for } u_{ij}\},$$
$$I_{jr} = \{i \in I \mid \text{condition set by rule } r \text{ holds for } u_{ij}\}.$$

In this setting, the only rule that does not fit in is the MNL, because in that case the utility is changed by the exponential, $U_{ij} = e^{v_{ij}}$. We have therefore omitted this from further discussion.

To complete the unified description of the rules discussed, we need to define the set S_{ir} for each rule r. This can be done as follows:

Binary: $S_{ib} = \arg\max_{j \in J} u_{ij}$

Probabilistic: $S_{ip} = J$

Multi-deterministic: $S_{im} = \{j \in J \mid j = \arg\max_{k \in C_l} u_{ik},\ l \in L\}$

Partially probabilistic: $S_{iP} = \{j \in J \mid u_{ij} \geq \overline{u}\}$

Pareto-Huff: $S_{iH} = P_i$

Brand preference: $S_{iB} = C_{B(i)}$

Covering-based: $S_{ic} = \{j \in J \mid d_{ij} \leq R\}$

Placing the above defined sets S_{ir} in (3.1) gives the specific formula for the customer selection rule r.

Now we are ready to reach the second aim of the paper, to construct a hybrid customer selection rule that combines the ones we have discussed before.

4. Hybrid customer selection rules

Most studies assume that all demand follows a particular choice rule. However, we know that we are all different, and some customers may follow one rule while others apply another. Therefore, at a demand point i, there may be customers who belong to each of the rules mentioned. Suppose we can estimate the proportion of customers who follow each rule. Let p_{ir} the proportion of customers at demand point i following rule r.

Thus, the market share is

$$ms_j = \sum_{i \in I} w_i \sum_{r \in R_{ij}} p_{ir} \frac{u_{ij}}{\sum\limits_{k \in S_{ir}} u_{ik}}, \tag{4.1}$$

where R_{ij} is the set of rules for which $j \in S_{ir}$.

In general, (4.1) is highly nonlinear and non-convex as it is composed of such functions. Note that although it is not highlighted in the formula, S_{ir} and R_{ij} are sets that depend on the location variables, and in many cases the objective function is not even continuous. However, in different settings it is easier to solve.

For the planar case, where the location of the new facility is continuous, either a geometric or interval branch and bound algorithm can be used, or heuristics, as in [6, 7, 9]. In the network setting, where locating on edges is possible, a special branch and bound method can be designed to solve such problems, see [1, 11] for similar works. These works also show that although the problem become difficult, it is still solvable for medium-size problems.

If there is only a discrete set of choices, but more than one facility is to be located, the problem leads to a Mixed Integer Nonlinear Programming problem, which might be linearized.

The surely tractable case for larger instances is when only one new facility is sought among a discrete set of choices. The simplified model for this case is shown next.

Suppose there is a discrete set of choices for the new facility, $f \in F$, and one new facility is being sought. In such a setting, the model is built using binary variables $x_f, f \in F$, with the value 1 if the location f is chosen, or 0 otherwise. Of course, $\sum_{f \in F} x_f = 1$ have to be added as a constraint, since only one facility is to be located. What makes this case easy is that for a given location f, one can directly compute all utilities and S_{ir} and R_{ij} sets, so the market share at the new facility can be written as

$$\begin{aligned} ms_f &= \sum_{i \in I} w_i \sum_{f \in F} \sum_{r \in R_{if}} p_{ir} \frac{u_{if} x_f}{u_{if} x_f + \sum\limits_{k \in S_{ir}} u_{ik}} \\ &= \sum_{i \in I} w_i \sum_{f \in F} x_f \sum_{r \in R_{if}} p_{ir} \frac{u_{if}}{u_{if} + \sum\limits_{k \in S_{ir}} u_{ik}} \end{aligned}$$

$$= \sum_{i \in I} w_i \sum_{f \in F} \tilde{u}_{if} x_f, \tag{4.2}$$

where the parameters $\tilde{u}_{if}$ are calculated as

$$\tilde{u}_{if} = \sum_{r \in R_{if}} p_{ir} \frac{u_{if}}{u_{if} + \sum\limits_{k \in S_{ir}} u_{ik}} \quad \forall i \in I, f \in F.$$

Note that we can omit x_f from the denominator in (4.2), since the whole fraction is directly zero if $x_f = 0$, and the fraction substituting x_f with 1 otherwise.

Now, with the calculated parameters, the problem becomes a rather easy to solve integer programming problem. The difficulty is rather to estimate the necessary data, that consist of estimating the different quality measures of the facilities, their importance, but also the proportion of customers belonging to each selection rule together with their corresponding details.

5. Summary

We have reviewed the most commonly used customer selection rules from the literature and found that most of them can be written in a similar form to the probabilistic selection rule, however they do not patronize all facilities.

After unifying the patronizing behaviours, we defined a hybrid selection rule, where it is assumed that at each demand point, customers may follow different selection rules. The hybrid rule is non-linear and non-convex, and as such is difficult to handle in general, although not worse than most of the individual rules.

Nevertheless, an integer programming model is given for the case where exactly one facility is to be located and there are discrete choices for the new facility. This model is easy to solve for even large data sets, however it is not trivial to collect all the needed data, as in all the customer selection rules described in the paper. Thus, as future work, it is planned to design a model which needs less amount of data to be estimated or simulated, but also to linearize the general model when more than one facility is to be located.

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