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Teaching numeral systems based on history in high school

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Abstract

In the first decade after the turn of the millennium, previous doubts about the inclusion of the history of mathematics in education also received more attention. Several researchers point to the difficulties of teachers enthusiastic on the topic, to the research methodological difficulties of such studies, and the need to increase the number of empirical researches. In addition to increasing the amount of such empirical evidence, this paper seeks to contribute to the continuously developing answers to the basic questions (what and how?) of integrating the history of mathematics into public education in the recent decades by presenting a given topic, the teaching of numeral systems based on history, and the results of the related surveys. In the course of our research, we examined the question of whether the use of the history of mathematics as a tool, as opposed to teaching by focusing solely on routine tasks, helps to fix the curriculum into the long-term memory.

Keywords: History of mathematics, teaching of numeral systems

AMS Subject Classification: 97A30, 97F30

1. Introduction

From the beginning of the 1960s, more and more researchers have turned to the use of the history of mathematics in education. In 1972, at the second International

Congress on Mathematical Education (ICME) in Exeter, an international research group, The international study group on the relationship between History and Pedagogy of Mathematics (HPM), was set up on the subject, which has regularly organized international conferences and published since then.

Research on the use of the history of mathematics in education has gained new momentum since the 1990s, and more and more researchers have given arguments about incorporating the history of mathematics into education (e.g., [2]). Although Lefebvre warns in his article summarizing this topic that "all forms of categorization carry risky and arbitrary parts" ([7], p. 24), by the 1990s, intensified efforts could be noticed regarding answering the questions why and how and categorizing the answers. For example, Fried [3] grouped the 15 arguments about the history of mathematics given by Fauvel [1] around a total of three themes: (1) making mathematics more human; (2) making mathematics more interesting, understandable, and approachable; (3) to allow a deeper insight into problems and problem-solving.

Jankvist's [5] article, written on this topic and quoted extensively, seeks to categorize the used methods (the how) and, separated from them, the arguments for use (the why). He classified the methods into three categories: the illumination approach, the modules approach, and the history-based approach. The first tries to spice up mathematics teaching mostly with isolated stories and anecdotes. This includes pictures that appear in the margins of textbooks as well as stories at the beginning or end of the chapters. The second category includes, for example, the study of a problem based on a topic in the history of mathematics and thus the way in which numeral systems are introduced in this paper. The third category means the presentation of the development, progression of the mathematical material covering a given part of the curriculum. Jankvist divided the answers to the question why into two categories. On the one hand, motivational factors that aid teaching and learning, on the other hand, tools that display the "soul" and development of mathematics, allowing the student not to see mathematics as a finished thing that "descended from heaven" in its perfection, axiomatically constructed.

In the first decade after the turn of the millennium, previous doubts about the inclusion of the history of mathematics in education also received more attention. Several researchers point to the difficulties of teachers enthusiastic on the topic (e.g., Fried [3]; Siu [9]), to the research methodological difficulties of such studies (e.g., Guillemette [4]), and the need to increase the number of empirical researches (e.g., Jankvist [6]).

In addition to increasing the amount of such empirical evidence, this paper seeks to contribute to the continuously developing answers to the basic questions (what and how?) of integrating the history of mathematics into public education in the recent decades by presenting a given topic, the teaching of numeral systems based on history, and the results of the related surveys.

2. The circumstances of the teaching experiment and the research question

The teaching experiment took place in Hungary, where public education is divided into two parts: an eight-grade primary school, after which students can continue their studies in a 3-year vocational school, a 4-year vocational grammar school teaching vocational and general subjects, finishing by graduation, or a 4-year grammar school (5-year for bilingual classes) teaching only general subjects, finishing by graduation, preparing for higher education. In high school, a class often has a profile of some kind, i.e., it studies the subjects related to that profile in a higher number of hours. During the admission procedure, most secondary schools select from the applying students on the basis of the admission points obtained on the uniform mathematics and Hungarian aptitude test worksheet.

The teaching experiment and then the related surveys took place in a grammar school in a big city in two consecutive school years in the ninth grade, in the first year in three groups of different profiles, and in the second year in two groups of the same profile.

During the experiment, the topic of numeral systems was taught to one group based on the history of mathematics, while to the control group, by focusing only on routine tasks, both studied for the same amount of time. This topic was not completely unknown to any of the groups. In primary school, all students had already learned about it in connection with the notion of sign-value and place-value notation when writing numbers.

In the course of our research, we examined the question of whether the use of the history of mathematics as a tool, as opposed to teaching by focusing solely on routine tasks, helps to fix the curriculum into the long-term memory.

Accordingly, we formulated the following principles for the curriculum constructed for the experimental groups:

- the inclusion of the history of mathematics in the classroom serves to teach the compulsory curriculum,
- as far as possible, the history of mathematics should be included in the teaching lesson as an integral part of the curriculum and not as a separate unit (for example, in the form of a student presentation or as mentioning interesting facts at the end of the lesson).

Following these principles, the lessons of the experimental group were about:

E1 To introduce the topic, to raise awareness, to describe that there are peoples (such as the Piraha of Amazon) whose language lacks the concept of numbers. And the question was, although numbers are important, whether it matters how we describe them?

E2 Introduction of the two characteristics of the numeral system we most commonly use, the decimal base and the place-value notation, by examples (see Figure 1).

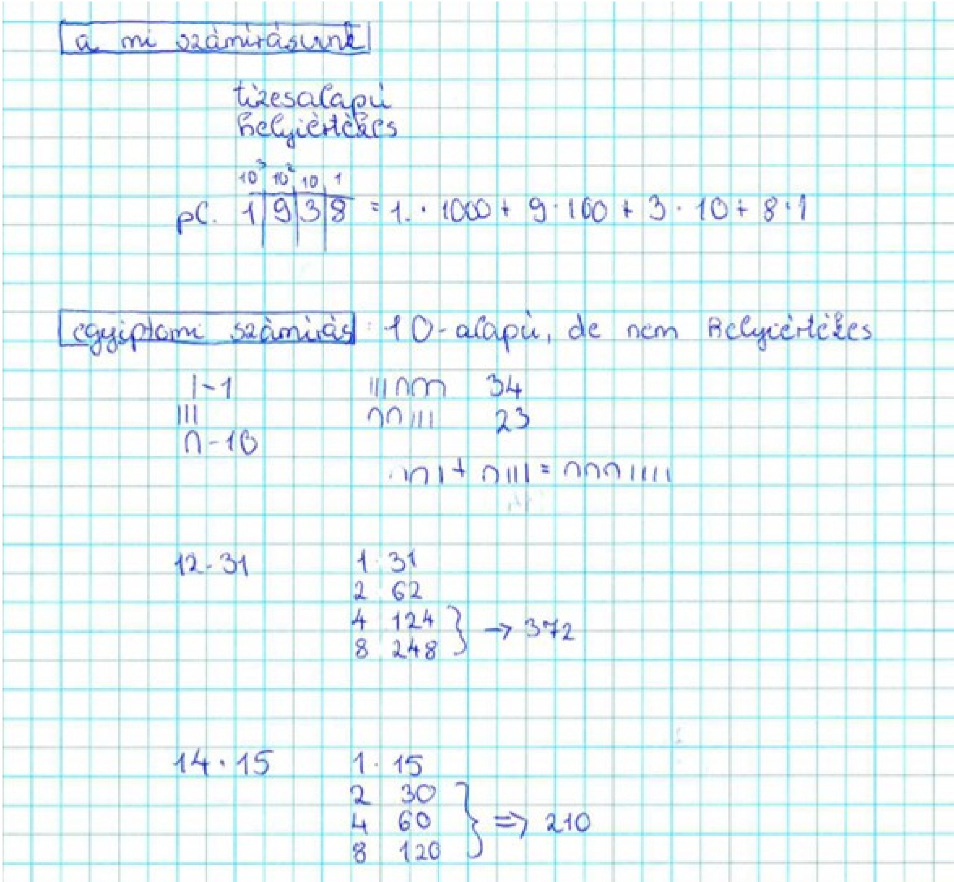

Figure 1. Our numeral system and the Egyptian numeral system (from the notebook of one of the students).

E3 A counterexample to one of these qualities: the ancient Egyptian numeral system is decimal but not place-valued. After a few examples, a description of addition and multiplication performed on integers. In the meantime, we highlight the following two problems: in theory, infinitely many different characters are needed to describe numbers, and sometimes a large number of characters are needed to describe numbers that are used in everyday life. The teacher examples presented were always followed by shorter or longer student work, so students had to independently multiply 14 by 15 in an Egyptian way (see Figure 1, "a mi számírásunk – our numeral system", "egyiptomi számírás – Egyptian numeral system", "tízes alapú – decimal". "helyiértékes – place-value").

E4 Example of the non-decimal and non-place-value numeral system: Roman numerals. This type of numeral system, while solving the previous two problems, raises another issue of the basic operations. Although students had already learned Roman numerals in the lower grades of elementary school, the number of knots and the logic of the system structure were revived.

E5 An example of the non-decimal but place-value numeral system. Noticing that the number of characters usable here is finite, and depending on the base number of the numeral system, a given number can be written in a longer or shorter form. Converting back and forth between decimal and other numeral systems having other bases. Adding in the non-decimal numeral system.

E6 Traces of numeral systems having other bases in our lives (watch, angles, notation of numbers in foreign languages.)

During the lesson, the teacher tried to guide the students to the key points by questions (e.g., How many characters are needed in the Egyptian numeral system to describe the number 798? Answer: 24.) or to bring to the surface the students' existing knowledge on this topic. (Are they familiar with any numeral system that is neither decimal nor place-value? Answer: Roman numeral system.) Because of the time constraints of the lessons, the teacher did not make the students rediscover the customs of other ages but introduced them (e.g., How did Egyptians multiply two integers?), thus maintaining their traditional "source of knowledge" role. The teacher in the control group viewed their own role as a teacher similarly.

In contrast, the lesson of the control group consisted of the following main sections.

K1 The two basic features of our numeral system are decimal and place-value, recalling the concepts learned in primary school (place value, sign value). (K1=E2)

K2 Examining what if the place values are not the powers of 10 but those of other positive integers other than 1. Converting back and forth between decimal and numeral systems having other bases. (See Figure 2) (This part corresponded to E5.)

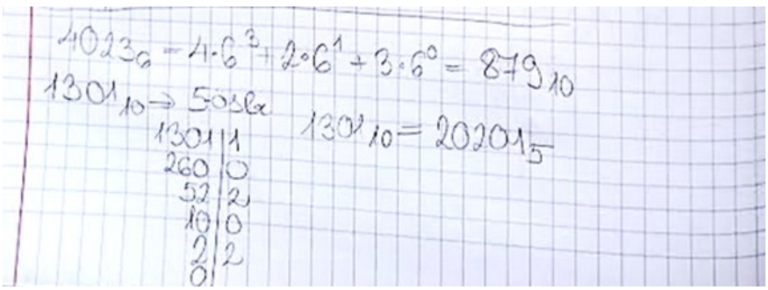

Figure 2. Converting between numeral system (from the notebook of a student from the control group).

3. Findings

The two teaching methods, followed by the related survey, were conducted in three groups with different profiles in the first year. Two of them had an experimental role, integrating historical elements, and the third was a control group. Among the experimental groups, group A consisted of students with a similar profile and similar ability to the control group; the other experimental group B consisted of students significantly different from the control group, with a weaker ability in mathematics. The latter group studied mathematics in French as a French bilingual group. The difference in their math skills is indicated by the average score on the high school admission. The data for the groups are given in Table 1. The lessons were held in a good atmosphere in all three groups; most students understood the curriculum, and no students indicated a problem during the homework check of the next lesson.

Table 1. Groups in the first experiment.

	Experimental group A	**Experimental group B**	**Control group**
Orientation of the group	Physics	French bilingual	Biology-Physics-Chemistry
Number of students in the group	14	13	14
Number of lessons in Math	4 per week	3 per week	4 per week
Average points on high school Math admission (50 points maximum)	37.00	31.00	35.56

The final tests written two to three weeks after the experimental lesson included a conversion task between numeral systems ("in both directions") for all three groups. In the case of these tests, there was no significant difference regarding the results of the students, at least 80% of the students in all three groups solved the task correctly.

However, two months after the experimental classes (including the two-week winter break in the meantime), at the beginning of a math class, students from all three groups were "unexpectedly" asked to answer the following two questions anonymously:

1. *Write the five-digit form of* 473.
2. *Write the decimal form of* 431_6.

The following statistics were compiled on the students' answers to the two tasks (Table 2). For Question 1, students in both experimental groups had more than 20% higher rates of correct answers than students in the control group. In Question 2, the proportion of correct respondents was 50% higher among the students of group A and 25% higher among the students of group B than in the control group.

Table 2. Summary of the students' answers (first experiment).

		Experimental group A	**Experimental group B**	**Control group**
From base 10 to base 5	**Number of correct answers**	13	12	10
	Number of incorrect answers	1	1	4
From base 6 to base 10	**Number of correct answers**	11	6	3
	Number of incorrect answers	3	7	11

In the second school year, we repeated the above experiment in groups with the same profile as the first year's control group (biology-chemistry-physics). In the experimental group, the average of the results of the 8th-grade math ability test was 35.06, while in the control group, it was 36.53, i.e., there was no significant difference between the math skills of the two groups.

As in the previous year, there was no problem either in the lessons or with the homework: More than 80% of the students solved the conversion tasks in a final survey written two weeks after the experimental lesson. As in the previous school year, both groups were "unexpectedly" given the next task two months after the experimental lesson at the beginning of a math class, which had to be answered anonymously:

1. *Write the binary form of* 345*.*

2. *Write the decimal form of* 1221_3*.*

The results of the two groups are summarized in Table 3. Question 1 had a 35%, Question 2 has 55% higher rate of correct answers among students in the experimental group than in the control group.

Table 3. Summary of the students' answers (second experiment).

		Experimental group	**Control group**
From base 10 to base 2	**Number of correct answers**	12	5
	Number of incorrect answers	5	9
From base 3 to base 10	**Number of correct answers**	13	3
	Number of incorrect answers	4	11

The percentage distribution of the summary of the responses of the students participating in the experiment during the two academic years two months after the lesson is shown in Table 4 by task type.

Table 4. Summary of the responses of the students participating in the two experiments.

		Experimental groups (44 students)	**Control group (28 students)**
Converting from decimal numeral system	**Proportion of correct answers**	84.10%	53.57%
	Proportion of incorrect answers	15.90%	46.43%
Converting to decimal numeral system	**Proportion of correct answers**	68.18%	21.43%
	Proportion of incorrect answers	31.82%	78.57%

4. Conclusion

At the beginning of our research, we sought to answer the question of whether teaching numeral systems in a historical framework is more helpful to fix information into the long-term memory than teaching focusing solely on solving routine tasks.

In response, we can state that although the control and experimental groups spent the same amount of time studying numeral systems in both school years, students who did not merely practice the same type of task again and again but learned about the topic more comprehensively, embedded in history, the tests written two months later definitely showed better results. It is in line with Revuz's idea, who stated as early as in the 1970s, “In the initial period of teaching, the most serious, almost irreparable damage can be done by replacing the true understanding with the mechanical practice of what has been learned”. ([8], p. 14.)

Examining the reasons, the question should be asked: what role did the history of mathematics play in the classes of the experimental groups?

- It provided a framework for the lesson that roughly followed the stations through which humanity came to the decimal, place-value numeral system used today.

- It provided a logical transition between the different numeral systems (e.g., the Roman numeral system addresses the problem of the number of characters required to describe numbers in the Egyptian numeral system).

- Mobilized the student's pre-existing knowledge (e.g., Roman numerals).

- In several cases, it gave a counterexample (e.g., decimal but not place-value system).
- It made numeral systems interesting and related to everyday life, while the other group mastered routine procedures that were impractical to life.
- It highlighted the fact that it is not the base number but the place-value notation that is important when performing basic operations (e.g., in the Egyptian decimal numeral system addition was completely different, but in the three-based place-value notation the principle of written addition does not differ from the decimal system), so this is the key concept of this topic. This was also manifested in the fact that during the surveys, the place value chart appears in the work of the students of the experimental groups, even in the case of the incorrect respondents (as shown in Figure 3 of the notebook of one of the students of the second-year experimental group).

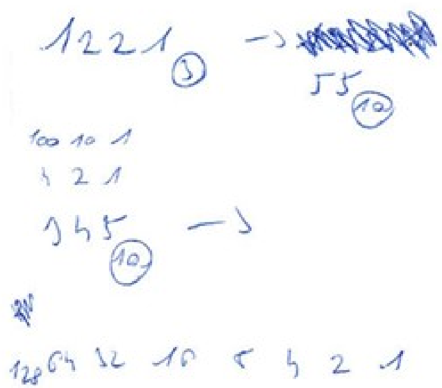

Figure 3. Appearance of the place-value chart in the incorrect answer.

The occurrence of all these may explain why the students of the experimental groups were able to recall what they had learned even after two months.

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