Annales Mathematicae et Informaticae
59 (2023) pp. 146–160
DOI: 10.33039/ami.2023.03.003
URL: https://ami.uni-eszterhazy.hu

The role of creation in the didactical traditions in Hungary*

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Abstract. In the didactics of mathematics, a lot of such research appeared in the past twenty-thirty years that consider both mathematics and mathematics education as a cultural activity [20]. In this approach, didactical texts and social space are carriers of beliefs created about mathematics education. Similarly, the investigation of beliefs has strengthened in the past two decades [33]. In our paper, we examine what relationship appears between the beliefs of the Hungarian didactic tradition existing in written texts or personal contacts, and those of Hungarian mathematics teachers. To carry it out, we use means of cultural history and a questionnaire.

Keywords: Belief, culture, literature, mathematical creation

AMS Subject Classification: 97A99

1. Introduction

At the very beginning, we intend to clarify some notions:

1. *Culture*
   We examine mathematical activity as a cultural activity. In the descriptive or scientific determinations, mainly the common patterns referring to groups of people are connected to the notion of culture [20]. On the one hand, culture exists in the space of behaviour with the help of tools and symbols, but parallelly with this, it can also be examined in the environment of thoughts

*This study was funded by the Research Program for Public Education Development of the Hungarian Academy of Sciences (KOZOKT2021-16).

Submitted: July 15, 2022
Accepted: March 23, 2023
Published online: April 26, 2023

and values. This ambivalence resulted in a kind of “cultural turn”, and 10-15 years ago the number of research considering mathematics education as a cultural activity increased. [20] Our article joins this direction; insofar it examines the presence of a thought – more exactly the thought of creation – connected to mathematical activity in historical and cultural context. The culture of didactics of mathematics can also be investigated; we take this as a starting point.

2. *Beliefs*
   Beliefs about mathematics and about mathematics education form a part of the culture of mathematics and its didactics. The research of teachers' knowledge, beliefs and attitudes started intensively in the last two decades, see [30] and got an overview by the Springer book [33]. According to Philipps' definition, a belief is the following: “Psychologically held understandings, premises, or propositions about the world that are thought to be true” [30]

3. *Creation in mathematics and in mathematics education*
   In mathematics, *creation* is possible at two different levels.

   (a) One of them happens when something new is discovered in an existing system. An existing, but a not yet recognised connection is found out. It can be a new statement, the proof of a new theorem or an old conjecture by linking two fields in a creative way, or proof given to an old theorem in a more elegant way, etc.

   (b) At a higher level: when completely new mathematical worlds, new universes are created as a result of a mathematician's creation process, such creations that show the nature of mathematics itself. Later on, we will show two examples to this (Bolyai's geometric creation and the significance of Gödel's role). This level is strongly connected with artistic creation as an act of creation; insofar the artist also creates an absolute world as a result of the process of creation.

According to us, the equivalent of the mathematical creation's notion can also be found in didactics. An example to this can be mathematics education by discovering which has a long tradition not only in Hungary. Here in Hungary this tradition is connected to the methodology of Tamás Varga [14, 37, 38], and also to the discovery method of Lajos Pósa [18]. In case of the discovery method, the result of the creation process can be experienced, the teacher gives such space to the student who – by going through it – can create own, individual results. The experience of creation is more strongly present in radical constructivism [4], in case of which such space is given to students where they similarly create some parts of knowledge by themselves, but more independently from the teacher. Constructivism based on this, merely has a tradition in Hungary. The connection between artistic creation and mathematics also appears in STEAM movement, which is a modern educational trend [24].

2. Question

With the help of the approaches and notions above, we examine a part of the comprehensive question, what kind of relationship exists in Hungary between the written tradition and personal tradition in didactics and in mathematics education. In what form does a thought – detected in written tradition – appear among the beliefs of teachers?

We intend to give answer to these questions, on the one hand, with the help of presenting such an effect history curve, in which we prove connections between mathematical texts and texts of mathematical didactics. On the other hand, having interpreted the outcomes of a questionnaire's particular items, we present in what way the thought appears – which is examined in the texts – among the teachers' beliefs, and from what sources the teachers' picture of mathematics derives.

3. The written tradition

The written tradition is interpreted as texts having mutual effect, as a net of (at times only loosely connected) texts referring to or quoting each other, and we examine this with the aid of the phenomenon of intertextuality [21]. Hungarian didactics also exists as a cultural activity. From this huge environment, we focus on the texts first, and on the fact as well, in what way the texts are in a dialogue with each other. If we approach the culture of mathematics and mathematics education from the viewpoint of beliefs and intellectual history, then one of the notions in the Hungarian tradition can be creation. Creation does not exclusively appear in Hungarian didactics as being equal to mathematical activity, but it also has a remarkable tradition among the conceptualisations regarding school mathematics ranging from the first grade of elementary school to university mathematics education (see Varga's reform and Rózsa Péter's work in the context of the teacher training). It has yet a connection with the artistic creation and the world of a literature. The proof of this can be seen by examining the history of one of Bolyai's sentences.

3.1. The sources of a sentence

The concept of creation flows along the history of European culture starting from the biblical Genesis creation ("God said"), through the creator divine Word (Word = God) which is known from the Gospel of John. To the history of the artistic creation belongs the myth of Prometheus, or by Shakespeare and Rousseau (see below). According to the conceptualisation of Sturm und Drang, in the Genieperiode, the creator is a genius, the self-contained artist is legalised to create a whole world with his own artistic power through breaking the rules [1]. The same power appears in János Bolyai's famous line in connection with mathematical creation. János Bolyai, who was the 19th century's greatest Hungarian mathematician and one of the most effective figures of mathematical history, reported on his revolutionary results in the field of hyperbolic geometry in a letter written to his father, Farkas Bolyai November 3rd 1823. The quoted sentences are taken from this.

"...from nothing I have created a new different world; all other things that I have sent to you are just a house of cards compared to a tower." [3]; (translation by Prékopa)

From the letter, we can draw two conclusions.

One of them is that J. Bolyai was aware of the scientific significance of hyperbolic geometry itself, and he wanted to let his father know about it by connecting creation and the creation of the new world with mathematical activity. This sentence refers to the parallelly existing geometries and mathematical worlds and to the genesis of these worlds at the same time.

The other conclusion is that the sentence referred to texts, which were known for both of them; what is more, they even talked about them with high probability. One of the possible sources is a contemporary Hungarian poem. One of the stanzas of Mihály Csokonai Vitéz's poem, titled To Solitude goes like this: [7]

> "In you, the poet's fancy flashes bright
> As rapid lightning in a murky night,
> While he creates new things by power of thought
> And fashions worlds undreamt-of out of nought."

It cannot be known if Bolyai knew the poem, but he doubtlessly knew Csokonai's first appearing biography. In the registry about the library of the Bolyais, the work of Márton Domby, who was the first biography writer of Csokonai, can be found. In the volume stands the following entry: "A book of János Bolyai. A gift from his father" [8]. Therefore, this book was read by János Bolyai, which can be almost surely declared. In the introduction, he encountered these lines:
"I would say first: These are unuseful materials [namely: the poems in the book], but secondly even for that reason are these brilliant, because you can see from them, *how the Genius creates a new world out of nothing...*" (italics mine). [9]

With this, Domby obviously referred to the poem titled To Solitude, with the difference that creation was interpreted as ability, not only by poets, but also by geniuses.

Bolyai's sentence can have other sources too, for instance, Shakespeare[1] or Rousseau[2], but Bolyai's sentence is so specific, and originally in Hungarian it equals almost literally with the two texts above to the extent, that it can be stated with high probability that Csokonai's poem, or rather Márton Domby's Csokonai-

[1] "The poet's eye, in a fine frenzy rolling,
Doth glance from heaven to earth, from earth to heaven,
And as imagination bodies forth
The forms of things unknown, the poet's pen
Turns them to shapes, and gives to airy nothing
A local habitation and a name."[35]

[2] "[...] the true genius that creates and produces anything from nothing." [34]

biography inspired it[3]. From both of the poems' context, it turns out that János Bolyai understood the significance of his own result, and he considered the role of the creator artist to be his own.

3.2. The mathematical-philosophical significance of Bolyai's work

As we have written in 1.c, creation in mathematics has different levels. Every new theorem, new connection, new theory is a creation. But it has a highest level when we create other type of mathematics compared to the one that we have in our culture. For 2000 years Euclid provided mathematics. Every new thing was adapted to this system until the appearance of János Bolyai. Then, this highest creation happened for the first time. Therefore, it is about the creation of a new world which repeated itself later, on the basis of Cantor, in set theory, but there along with preliminaries, as Kurt Gödel had already proved his famous incompleteness theorem by that time in 1931, which provided an interpretational frame to the achievement of Bolyai [12, 17]. The theorem plainly means that in any axiom system in which the infinite appears, so, at least the natural numbers are included, such a statement can be formulated that can neither be proved nor denied in the given system. Namely, an unexpected, new situation has been created, as up to that time something was either true or false, but from that point on there are things about which free decision is possible as the statement can neither be deduced nor rejected. At this point, a new way of creation appears, a new kind of mathematics can be constructed to the axiom system by attaching its statement or its rejection to it (one of them is usually the *old* world and the other one is the new one). Therefore, mathematics is not universal; there are more mathematics next to each other. In addition, János Bolyai, respectively Lobachevsky at around the same time, found this first case of an *undecided theorem*.

Half a century after this came the hypothesis of Cantor, being at the same time Hilbert's first problem in 1900, among the range of problems to be urgently solved at the beginning of the XX. Century [16]. *Is there cardinality between the uncountable and the continuum? Cantor's hypothesis was that there is not.*

Finally, to this, an unexpected answer arrived. In 1938 Gödel proved with the help of the sets, the so called *constructive sets of Gödel* – named after him – that Cantor's hypothesis cannot be denied [10, 11]. Paul Cohen admitted that

[3] In Hungarian the similarity of the three texts is more visible:
Bolyai's sentence: "*A semmiből egy új, más világot teremtettem.*"
Csokonai's poem:

> Tebenned úgy csap a poéta széjjel,
> Mint a sebes villám setétes éjjel;
> *Midőn teremt új dolgokat*
> *S a semmiből világokat.*

and Domby's sentence: "Erre én is azt mondom először, hogy igen is: Tanúság nélkül való, és haszontalan Matériák ezek [értsd: a könyvben közölt versek]: de másodszor azt, hogy éppen ezért remekek ezek, mivel ezekből látod, *miképpen teremt a' Genie semmiből is Világokat...*"

its rejection cannot be denied either, having done that with the method of forcing created by him, meaning that numerosity can exist between the two [6]. With this, it turned out that in the axiom system of Zermelo–Frankel the continuum hypothesis can neither be proved nor denied, therefore two different set theories can be imagined, according to our decision (like during the time of Bolyai the Eucledian and the non-Eucledian geometry). Thus, we have found *the second case of an undecided theorem*. Ever since not another one's fact of existence has reached us.

From then on, in case of a statement there are three opportunities instead of the – up to this time known – two. In this case, mathematics goes on in both directions, in one of them there will be an axiom, in the other one its rejection will be the axiom, so, these are two new worlds. This revelation was revolutionary, and it shook the – up to that time formed – picture about mathematics radically. However, it merely has had an effect on school mathematics until now. We think, it can have a positive effect if teachers themselves have a deeper understanding of this situation and also if they accept it.

3.3. The history of effect of the sentence

In Rózsa Péter's Playing with Infinity [29] two passages deal with Bolyai's results and works. In one of the passages, the following is circumscribed as the two most important and essential attributes of the geometry of Bolyai: 1) The *opportunity of choice*. Going towards (in those days) the most modern results of logic, the text is about the choices of axiomatisation, and taking the fact into consideration that the author formulates statements and opinions consequently on mathematics in general throughout the book, the notion of choice can grow into the symbol of *freedom* referring to mathematics in general. 2) Even if experienced reality can motivate the birth of mathematical notions, it is not identical with mathematical reality.

The other passage is shorter but equally important from the point of the examination of the text's cultural context. The following passage brings the connection of mathematical creation on surface:

"What is all this really about? Man created the natural number system for his own purposes, it is his own creation; it serves the purposes of counting and the purposes of the operations arising out of counting. Nevertheless, once created, he has no further power over it. The natural number series exists; it has acquired an independent existence. No more alterations can be made; it has its own laws and its own peculiar properties, properties such as man never even dreamed of when he created it. The sorcerer's apprentice stands in utter amazement before the spirits he has raised. The mathematician 'creates a new world out of nothing' and then this world gets hold of him with its mysterious, unexpected regularities. He is no longer a creator but a seeker; he seeks the secrets and relationships of the world, which he has raised." [29]

In this passage, the activity of the creator (inventor) and the researcher (discoverer) is connected on the basis of the famous sentence quoted from János Bolyai.

To Goethe's poem, titled The Sorcerer's Apprentice, a whole chapter's title refers in Rózsa Péter's text. The poem itself tells a story, in which the sorcerer's apprentice – left alone – tries to bring the broom to life, he succeeds, but the broom gets loose and does not obey to him, but the spell supposed to reverse this act is not known by the apprentice, that is why, in the end, the apprentice has to be rescued by the sorcerer because everything would be flooded if the broom did not obey again.

Rózsa Péter says about the apprentice: "The sorcerer's apprentice stands in utter amazement before the spirits he has raised". Here an important moment's dramatization happens from a psychological point of view by paralleling the mathematician with the sorcerer's apprentice. With this parallel, Rózsa Péter depicts the moment when a mathematical creation is born. If we compare this step with the text of the Goethe-poem, then we can state the following: Mathematical creation is in such a relation to a person – to the creator itself – as a sorcerer's apprentice to a ghost. It is about a larger existence than the apprentice here, to whom laws do not apply, even though the sorcerer's apprentice himself created them. These mathematical creations can be monumental things like hyperbolic geometry, or much less monumental, but yet similar creations in nature like natural numbers that Rózsa Péter mentions, too. The common feature that makes the basis of the parallel is the fact that the creator does not have power over the creature. If we observe this simile from the viewpoint of knowledge and cognition, then it sheds a light on the fact that the construction of mathematical knowledge is an act of invention and discovery at the same time, but also on the other fact that mathematics is not a closed and finite set of knowledge.

We are convinced that this textual tradition contributed to the history of mathematics education in Hungary. From this symbolic sentence of the history of mathematics, and from the context of this sentence by Bolyai, some specific features of mathematics education in the Hungarian culture of didactics can be detected:
The concept of creation is a key part of mathematical activity. But it is not only connected to mathematical activity, but also to school mathematics from the beginning of the second half of the 20th century. The movement of complex mathematics education in the 20th century, based on T. Varga's complex mathematics education experiment belongs to the guided discovery method in mathematics education and partially also to the inquiry based mathematics education.

Erika Jakucs formulated the following sentences in the documentary film of Tamás Varga in 2019 [37, 38]:
"Teaching math is like a game field. Like creating a bord game. We create the aboard and the figures, and set down the rules, and then, we have to play by these rules. Still, math opens many areas, where we can figure out what the rules are, what type of objects do we have to use for these rules to follow through, and we must follow through these rules. Therefore in math, we can create small enclosed worlds or systems. There we can learn that if we agree to certain rules – and that's the key word – then we must adhere to these rules. Alternatively, agree to certain amendments. For me this is important, because all natural sciences work with

given materials, while math creates its own resource materials for any situations. You have a greater freedom."

Tamás Varga was one of the organisers of the aspirations of mathematics education in the previous century, and was one leading figure of an extensive international school reform [13] One of his principles is that creation during mathematical activity can be recognised starting from early childhood. The curriculum of the reform and the methodological materials connected to it were created with his collaborators in a way that this principle was all along taken into consideration. Of course, it does not mean the creation of the new mathematical world, but the understanding of it within the environment of school mathematics, mainly among the frames of conceptualisation taking place during games, which were based on experience [15].

In Bolyai's sentence – according to the concept of Romanticism – only the chosen ones and geniuses are blessed with the ability of creation who are separated from the public. By the 20th century, the concept of creation was deprived of this romantic genius-cult and one of the key points of Tamás Varga's reform was that he did not think in elite education but he intended to make mathematics and the possibility of experiencing mathematical creation within the frames of public education available for everyone [5].

Rózsa Péter writes about the fact in Playing with Infinity (first in [28]) that there are no two cultures (humanities and science) but art and mathematics are related. With this, she refers to the text written by C. P. Snow [32, 36]. Rózsa Péter collaborated with Tamás Varga, and this approach to culture can be felt on the works by Tamás Varga, too.

3.4. Some phenomena that can be indirectly connected to this tradition

István Lénárt created the so-called Lénárt Sphere, which is suitable for visualising and teaching spherical and hyperbolic geometry in the junior section of elementary school [22].

With this teaching aid and the methodological construction connected to it, Lénárt made not only Bolyai's elementary steps of his mathematical results ostensive, but also the historical significance of mathematical result itself. Students can experience the opportunity of the *freedom of choice* between geometrical worlds.

In the environment of Hungarian mathematical talent development, which is known inter-nationally, creation appears to be a key part of mathematical activity. In this field, Lajos Pósa is the leading person in Hungary. He and his foundation organise this activity in a non-regular way, but in form of weekend or summer camps. (About the theoretical background see Dániel Katona's works, for example [18], whose PhD-research is about the Pósa-method.)

Not necessarily the Pósa-idea of talent management should be thought about to detect the notion of creation. About the creation of real numbers using geometry, the number line, see yet [25, 26] and similarly about the creation of rational numbers beginning from fractions see [27].

The Hungarian project is the carrier of the written tradition from a cultural point of view (led by Ö. Vancsó) which deals with the idea and background of the Complex Mathematics Education of T. Varga, which has rooted in this tradition in Hungary[4].

4. Tradition based on personal contacts

On examining the history of Tamás Varga's reform, one can conclude that the key element of the reform's spreading was engagement, which happened with the help of personal contacts. Many reports can be read how it happened.

Anna Kiss says in an interview about her own professional development as she talks about her participation in someone else's lesson as a listener: "Then it became obvious to me that I have to breathe from the air that determines the lesson, one has to experience it in one piece, how these discovery processes happen, and after that I have to find out what I can create in a way that is mine, too." (The source of the text is a book being about to be published.)

Thus, the functioning of the written tradition happens differently in a different environment. Below, I intend to report on a questionnaire's partial results. More specifically on those results that are in connection with the written and personally transmitted tradition and with the – in them appearing – notion of creation.

5. Questionnaire

This year, we examined a segment of teachers' beliefs related to mathematics as a school subject through an online questionnaire. We collected questions from a previous project LEMA [23] as a starting point, rephrased some of them and added further questions as well.

Now, we will only focus on some questions from the questionnaire, only on those ones, which give us information about the inquiry if the teachers' beliefs were influenced more by the written or the personal tradition, and also about the fact whether the notions of creation and freedom can be connected to school mathematics. The whole questionnaire can be the basis for further research[5].

[4] An international conference (Varga 100) was organised by this project in order to introduce that Varga's main ideas are also relevant today. You can read about it in two special issues of Teaching Mathematics and Computer Science (TMCS 2020/18.3). Another goal of this conference was to present the Legacy of T. Varga and to position it in the modern didactical trends. See K. Gosztonyi's plenary [14] and the panel plenary led by M. Artigue [2].

[5] A big and significant survey of this kind is the one carried out by the MTA Working Group on Mathematics in Public Education [19]. It worked with a large sample (4257 answers). The goal of this research was to find the biggest problems in mathematics education in Hungary. We did not want to draw up a complete belief map, as this can only be done through a more comprehensive and detailed test.

5.1. The sample

777 teachers took part in the questionnaire[6]. The distribution of participants by school type was as follows: 57 from lower primary (1–4. classes), 472 from upper primary school (5–8. classes), 64 from eight-year high school (5–12. classes), 79 from six-year high school (6–12. classes), 200 from four years high school (9–12. classes), 28 from vocational secondary school and 24 from other type (there are teachers who are employed in more than one schools). We do not have exact data, so we do not know how representative our questionnaire is, nevertheless, the total number of respondents is high, compared to previous research of this kind.

5.2. Questions

However, we asked not only closed-ended questions, but also open-ended questions where teachers could elaborate on the subject. This was important because we received several responses that were not closely related to the questions we asked, but clearly showed teachers' related areas of interest. Teachers were asked the following series of questions:

5.2.1. Introduction

"*Title: RESEARCH – subject-related questionnaire*
In the research below, we examine with Dr. Ödön Vancsó what picture exists in the head of practising teachers about school mathematics. It would help our work to a great extent if possibly the most teachers could fill in this form throughout the country, therefore, we are curious about your opinion, too..."

We had three questions about the years spent as a teacher (practice-time), where she/he got the university-degree and about other subjects.

Further questions and answers were as follows:

5.2.2. Questions on a five-point scale

"*At each question answers on a scale from 1 to 5 can be given.*
1 = I don't agree at all ... 5 = I completely agree."

For a significant number of questions, it is clear that on a five-point scale, answers are either "*agree*" or "*disagree*". Based on this data, we could say that there is a perception among the participants about the subject of mathematics. According to them:

1. Mathematics is an area where students can discover things.

2. In problemsolving, you don't need to know the one and only solution, you can experience choice and freedom.

[6] We would like to thank Gabriella Hajnal, Dániel Bebrevszky and Róza Hitérné Erdős for their help in sending out the questionnaire and all the participants for answering it.

Table 1

Question	1	2	3	4	5
If students get to grips with mathematical problems, they can often discover something new (connections and rules).	3 (0,4%)	25 (3,2%)	90 (11,6%)	310 (39,9%)	349 (44,9%)
In order to solve a mathematical task, one has to know the one and only correct procedure.	309 (39,8%)	237 (30,5%)	171 (22%)	45 (5,8%)	15 (1,9%)
In mathematics one finds the experience of freedom.	8(1%)	32 (4%)	163 (21%)	321 (41,3%)	253 (32,6%)
Every student can create or recreate parts of mathematics.	53 (6,8%)	132 (17%)	261 (33,6%)	226 (29,1%)	105 (13,5%)

5.2.3. Open-ended questions

The questionnaire also asked, where the respondents studied and how was their image of mathematics formed. Connections, significant teachers, personal student and teaching experiences dominated most in the formation of their image of mathematics.

"*Please write down some factors that influenced your opinion on mathematics as a school subject. These can be reading or other kinds of experiences, people, institutions, etc.*"

"*Other. In case of any other comments regarding the questionnaire that you would like us to know about, and that do not fit into any categories, please share with us here.*"

5.2.4. Results

1. When asked about creation, we did get the following answers.

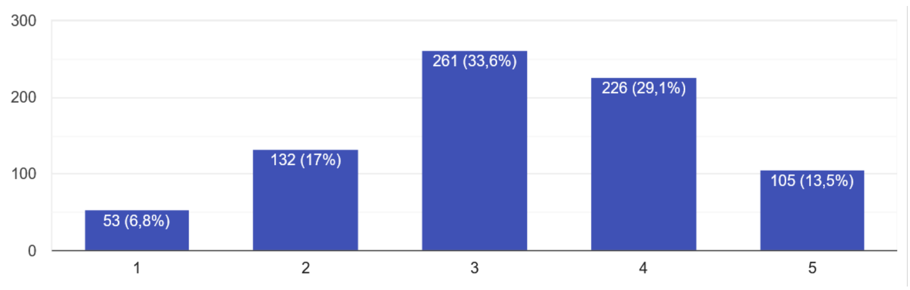

Figure 1. "Every student can create or recreate parts of mathematics."

The answers were in the middle of the range, but we can say that creation as a belief is part of school mathematics in Hungary.

2. In the cultural tradition of mathematics education, there is a need for creation and artistic activity in relation to mathematics and mathematics in schools, but the practice seems to show something else. Creation is present, but not as prominently as one might expect based on the work of Tamás Varga and the leading didacticians and teachers of the 20th century.

3. Reading materials (books, journals) were much less frequent among responses, thus basic ideas in written didactic tradition influenced the teachers' image of mathematics through personal experiences. The written tradition and the transmission of practical experience are in harmony.

4. In conclusion (also based on the textual responses) we can say, that on the one hand there is an idyllic image of a school mathematics (that teachers consider to be good and that exists in their dreams as a content- and methodological environment). On the other hand, there is the reality that is much more prosaic and much less in line with teachers' image of mathematics. The tension between the two is evident from the textual responses.

To underline the last conclusion, we would like to quote two textual answers:

- "*In Hungarian public education, mathematics is taught in very heterogeneous groups. It would be nice if there were opportunities for discovery mathematics when the amount of practice added at home is negligible. (I teach in a high school, and the standards are going down there too. The amount of practice students do at home is not enough. But what can a district school say against the university's practise high schools in Budapest!*"
- "*I face it every day: classes of 33-36 students, maths lessons [45 minutes] for the whole class, 3 per week (this is in grades 9-10, before that there are smaller groups, but the number of lessons is still 3 per week), 24-26 teaching hours per week, plus free tutoring, specialised classes. Nevertheless, my enthusiasm is still there:) We compete in lessons, workshops, cutting, metalworking, playing free play...*"

6. Conclusions

Therefore, the philological idea that Bolyai's sentence was inspired by a poem or by a poetical biography, belongs to the history of the connection between creation and mathematics. With all its antecedents and effects, it can be proof for the fact that the *need for creation as a belief* is part of the Hungarian written didactic tradition. This belief is worth being taken into consideration when a current phenomenon of mathematics education is to be interpreted.

Based on the results of our questionnaire we can say that the concept of *creation* and another concept of the tradition, the *experience of freedom* is present among the beliefs of the mathematics teachers in Hungary. However, it must also be taken into account that reality and the day-to-day problems of school mathematics do not allow the basic ideas rooted in the tradition to develop sufficiently.

Nevertheless, this tension can be mildened by focusing on the common features existing in the tradition and among the beliefs. The personal experience seems to be the main source of the mathematics teachers' image of mathematics.

Acknowledgements. We would like to thank everyone who completed the questionnaire.

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