*Annales Mathematicae et Informaticae*
***37*** *(2010) pp. 199–210*
http://ami.ektf.hu

# Spatial Ability, Descriptive Geometry and Dynamic Geometry Systems

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*Submitted 25 November 2009; Accepted 22 August 2010*

**Abstract**

Dynamic Geometry Systems allow new opportunities for the teaching of geometry and descriptive geometry. These systems make possible to create dynamic drawings quickly and flexibly. In the University of Debrecen Faculty of Engineering we executed a controlgrouped developing research for two years, one of them was at Descriptive geometry with participating first year full-time Mechanical engineer students and the other one was at Technical representation practice, in two-two practical groups, for trying out a teaching-learning strategy. We taught one of the groups with the help of Dynamic Geometry System, the other one traditionally, with the paper-and-pencil method. In this paper, I report on our experiences of this course.

*Keywords:* Spatial ability, descriptive geometry, dynamic geometry.

## 1. Introduction

Descriptive Geometry provides training for students' intellectual capacity for spatial perception and it is therefore important for all engineers, physicians and natural scientists. "Descriptive Geometry is a method to study 3D geometry through 2D images thus offering insight into structure and metrical properties of spatial objects, processes and principles" [19]. Moreover some basic differential-geometric properties of curves and surfaces and some analytic geometry are included and one aim is also to develop the students' problem solving ability [20].

The most important ability in working with Descriptive Geometry are the ability to perform operations on the basis of definitions and the spatial ability. We get most of our knowledge in a visual way so it is very important for us how much we are aware of the language of vision.

Spatial ability for engineering students is very important, which decides of the future career. These abilities are not determined genetically, but rather a result of a long learning process. The definition of spatial ability according to Séra and his colleagues [18] "the ability of solving spatial problems by using the perception of two and three dimensional shapes and the understanding of the perceived information and relations" - relying on the ideas of Haanstra and others [4].

Séra and his colleagues [18] are approaching the spatial problems from the side of the activity. The types of exercises:

- projection illustration and projection reading: establishing and drawing two dimensional projection pictures of three dimensional configurations;
- reconstruction: creating the axonometric image of an object based on projection images;
- the transparency of the structure: developing the inner expressive image through visualizing relations and proportions;
- two-dimensional visual spatial conception: the imaginary cutting up and piecing together of two-dimensional figures;
- the recognition and visualization of a spatial figure: the identification and visualization of the object and its position based on incomplete visual information;
- recognition and combination of the cohesive parts of three-dimensional figures: the recognition and combination of the cohesive parts of simple spatial figures that were cut into two or more pieces with the help of their axonometric drawings;
- imaginary rotation of a three-dimensional figure: the identification of the figure with the help of its images depicted from two different viewpoints by the manipulation of mental representations;
- imaginary manipulation of an object: the imaginary following of the phases of the objective activity;
- spatial constructional ability: the interpretation of the position of three-dimensional configurations correlated to each other based on the manipulation of the spatial representations;
- dynamic vision: the imaginary following of the motion of the sections of spatial configuration.

The link between engineering students' spatial ability and their success in a range of engineering courses is very important. Mental Cutting Test (MCT) is one of the most widely used evaluation method for spatial abilities. Németh and Hoffmann [14] presented an analysis of MCT results of first-year engineering students, with emphasis on gender differences. They used the classical MCT test for first-year engineering students of Szent István University. Németh, Sörös and Hoffmann [15] attempted to find possible reasons of gender difference, concluding, that typical mistakes play central role in it. They show typical mistakes can be one of the possible reasons, since female students made typical mistakes in some cases more frequently than males. In accordance with the international experiences, they observed relevant improvement after descriptive geometry courses. Williams and his colleagues' paper [24] and others [10] report on research into the spatial abilities of engineering students, too. MCT and similar tests have been widely studied in the following papers: [3, 5, 17, 21, 22, 23].

One of the programs, that supports computer-aided descriptive geometry was developed by a Hungarian expert and helps the teacher to explain the theory and practice of the Monge projection, the reconstruction of the spatial objects in the mind and, with the help of interactive feature, to understand spatial relationships [8]. Designs can be saved in BMP format.

At the University of Debrecen, Faculty of Engineering, we can experience that the basic studies have their difficulties: there are huge differences among the pre-education level of the students, the number of lessons is continuously decreasing and education becomes multitudinous. In our college, full time engineer students have a 2 hour seminar and a 2 or 1 hour lecture in every course from descriptive geometry. During that period of time they should pick up the elements of Monge-projection to the interpenetration of flat bodies and the curvilinear surfaces. (The syllabus differs according to their major.)

The interest, the pre-knowledge and motivation of the students are very different. One of the problems of the traditional teaching is that these problems can not be easily managed. But the use of computer tools makes it possible that each and every student can proceed in his own speed, so they do not lag behind and they do not get bored. The student can plan his/her own pace of learning and the speed of development.

This article reports about our experiences and results of descriptive geometry course.

## 2. Tasks with Dynamic Geometry Systems

Literature suggests that Dynamic Geometry Systems (DGS) is a valuable tool to teach geometry in schools [1, 2, 6, 7, 9, 16]. These systems are not only complement static geometrical figures, but also the software stores construction steps throughout its use and objects can be treated as dynamic figures. In this way when parts of figures are altered then this change also modify the entire figure structure. Thus, students can follow how elements of figures are built on one another.

Laborde [10] classified these tasks according to their role that the designer of the task attributes to Cabri (another type of DGS) and to the expected degree of change. The four type of roles:

- DGS is used mainly as a facilitating material, while aspects of the task are not changed conceptually.

Our example: Figure 1 shows the construction of a worksheet and Figure 2 shows the right solution. (Figure 1 and Figure 2 - Created with *Cinderella.*) (Interactive worksheet 1 - in our phrasing.)

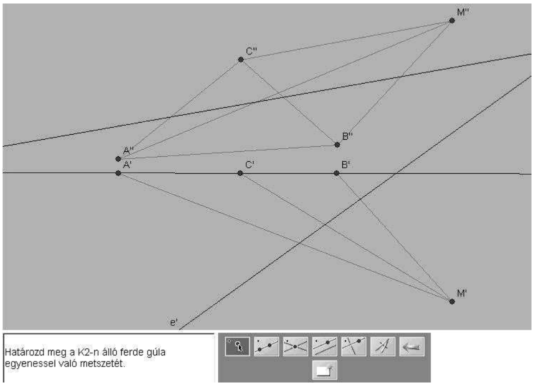


Figure 1: Construction of a worksheet

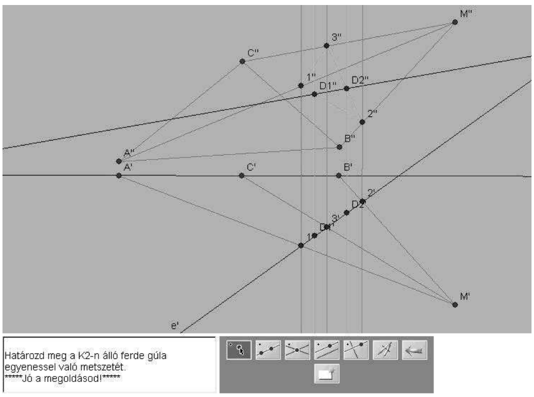


Figure 2: The right solution

- The task itself takes its meaning from DGS (for example Black-Box tasks), with DGS construction tools and dynamic features.

  Our example is Pyramid's plane section. (Figure 3 - Created with *Cinderella.*) (Interactive worksheet 2 - in our phrasing.)

  The pictures of the Figure 4 show the use of the program's dynamic features in descriptive geometry. On the left side moving the point P to the right side's projection picture we can trace back the representation of the picture

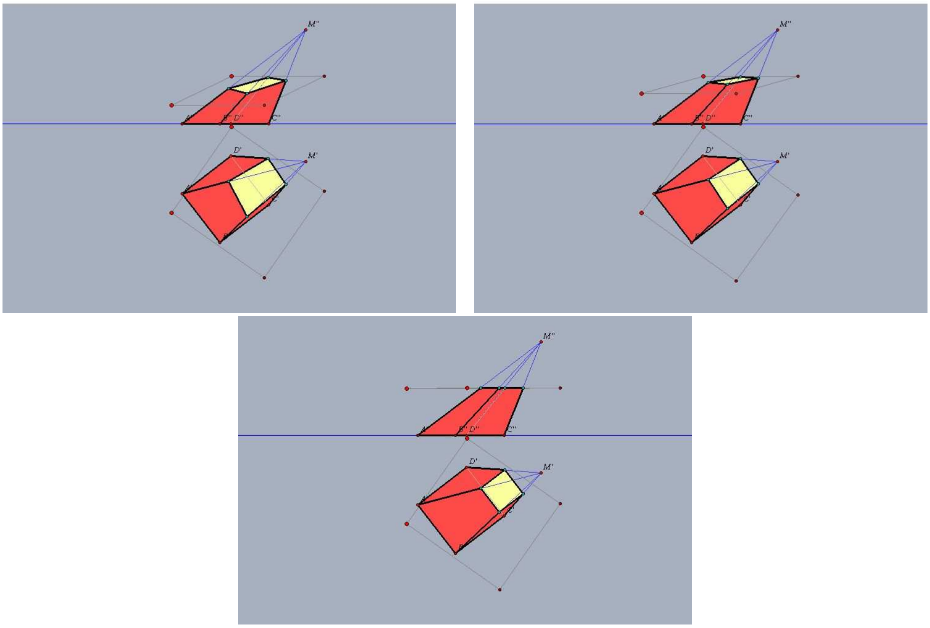

Figure 3: Pyramid's plane section

if our point is at the I., II., III. or IV. spatial quarter. (Figure 4 - Created with *GeoGebra*.)

- DGS is used as a visual amplifier (static fugures).

  Our example: There are two planes, the first with the ABC triangle, the second with the 123 triangle, from which we cut out the 456 triangle. Construct the intersection of the two planes. To state the visibility you should consider the holes. (Figure 5 - Created with *Cinderella*.)

- DGS is supposed to modify the solving strategies of the task, with some construction tools of DGS.

So by means of movement it can be observed how figures are constructed upon each other, as well as the construction process itself [7]. If a constructed figure in the drag mode does not keep the shape that was expected, it means that the construction process must be wrong [10]. Looking at how students used dragging provided an insight into their cognitive processes [1].

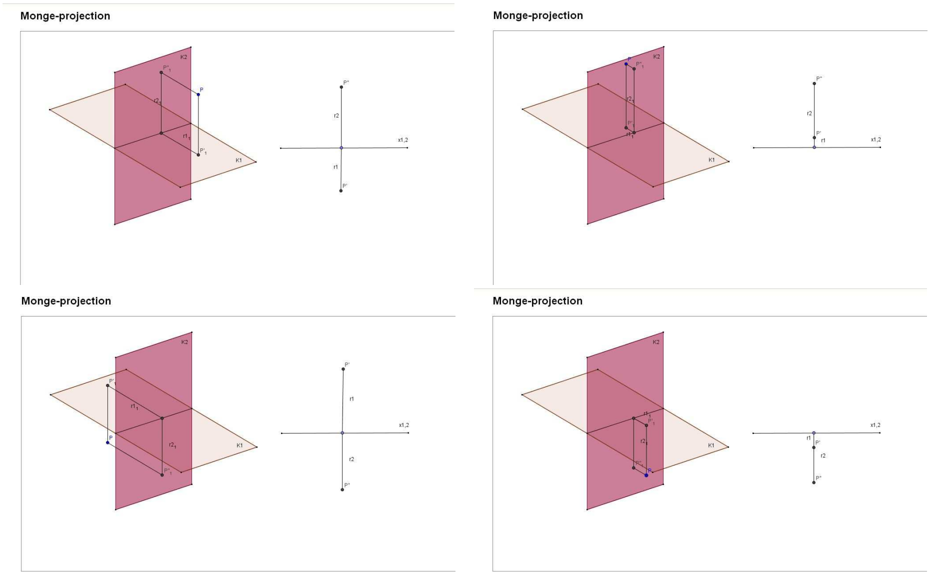


Figure 4: Representation of a point

# 3. Experiments

In the University of Debrecen Faculty of Engineering we executed a controlgrouped developing research in two semesters, it was at Descriptive geometry with participating first year full-time Mechanical engineer students, for trying out a teaching-learning strategy. We taught one of the groups with the help of DGS, the other one traditionally, with the paper-and-pencil method. We carried out the educational research with 80 first year full-time Mechanical engineer students at Descriptive geometry practice, in two-two practical groups.

In the University of Debrecen, Faculty of Engineering the students selected for the engineering programme acquire the basics of the Descriptive geometry - the elements of the Monge projection - in the course of a 2-hour lecture and a 2-hour seminar each week, which they use later in their professional subjects. From the two seminars we held, one group worked with DGS and interactive whiteboard, while the other group did constructions in the traditional way with paper and pencil for two years. The tests were paper-and-pencil tests, even for members of the group that had been working with the computers. The tasks are the traditional paper and pencil tasks. It does not contain theoretical question but practical ones.

Our goals:

- To meet the curriculum requirements.
- Increasing the understanding of the Descriptive geometry.

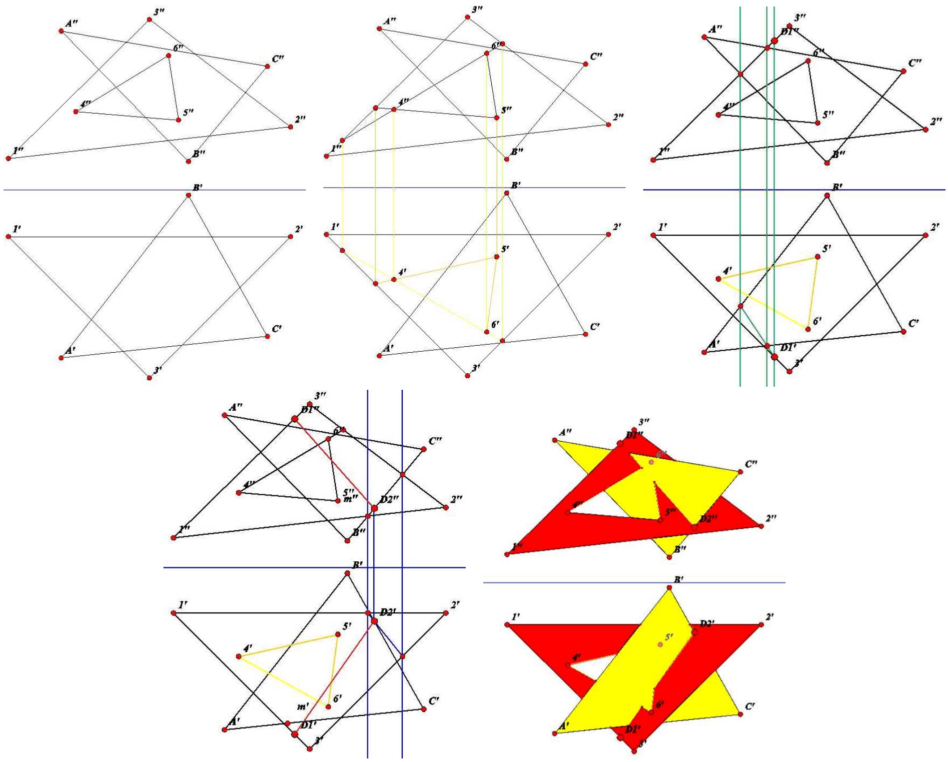


Figure 5: Intersection of planes

- To meet the mass education demands.
- To get prepared one part of our students to get into the university level.

During the semester the lecture went on without using a computer. The two seminar groups had only one computer room, so the first group worked with DGS while the second worked with the traditional paper and pencil methods. We paid special attention to make sure that all the two groups have the same tasks and they got the same paper and pencil homework. The difference is in the drawing opportunities of the program. The solution strategies of both tasks do not differ essentially. These are the peculiarities of the first type of exercises made by Laborde [10]. We tried to organize the practises in such a way that neither approach-whether teaching aided by the DGS, or teaching using paper-and-pencil had an advantage. Thus we hoped to achieve reliable measurements of relative ability. DGS that we chose for practice is able to save all the constructions as an interactive webpage. So there is an opportunity to make worksheets to practice constructions. The teacher can adjust the set of starting objects and desired objects. After that he/she can save the task with a limited use of geometric tools. Since we do not have to adjust the proceedings of the solution, we just have to adjust the set of the desired objects, so the program accepts more than one approach to the good solution. You can attach guidance or construct help to the worksheet. There is no need to install the program, to make the interactive worksheet; you should only attach a special file to the web page besides the tasks. Using this opportunity the students could work online on the seminars. To the education of the Descriptive geometry that is using DGS we made a webpage made up of Cinderella worksheets, involves the material of the practises. We tried out this curriculum system throughout two years and we continuously examined its efficiency. The curriculum system processed by us, which was suitable for teaching the Descriptive geometry according to the experiences of the 2004/2005 school year we modified and revised it in the 2006/2007 school year.

In the preliminary phase the students' levels of knowledge was examined by measurement of spatial ability. At the beginning of the semesters we examined whether there is a significant difference between the spatial ability of students in their preliminary basic knowledge of descriptive geometry. The measurement of the students' preliminary knowledge took place in the first teaching week. The exercises can be categorised under the following headings [18]:

- imaginary manipulation of an object,
- imaginary rotation of a solid,
- projection description and projection reading,
- reconstruction.

By the preliminary survey it can be seen that the two groups achieved nearly the same. The results of this test are presented in [11].

## 4. Results

To measure the efficiency of the teaching-learning process during the semester, there were two tests and one delayed test, consisting of practical exercises, which was taken by the students four months after the semester; all of which we rated with a score [12, 13]. The comparative survey of the results is based on these tests.

The test of the computer group was more punctual, a little more precise and it was better in both years. But the determination of transparency in 2004 was wrong more often by them than by the students of the paper-and-pencil group. In 2006 from learning this we paid larger attention for practising the determination of transparency in the computer group. Based on both tests we can say that the computer-aided group carried out the acquirement of the legally given educational requirement better than the control group. Figure 6 shows the result of the two tests and the delayed test.

In the traditional, paper-and-pencil first and second tests can be observed that the students of the computer-aided group perform better than the students of the

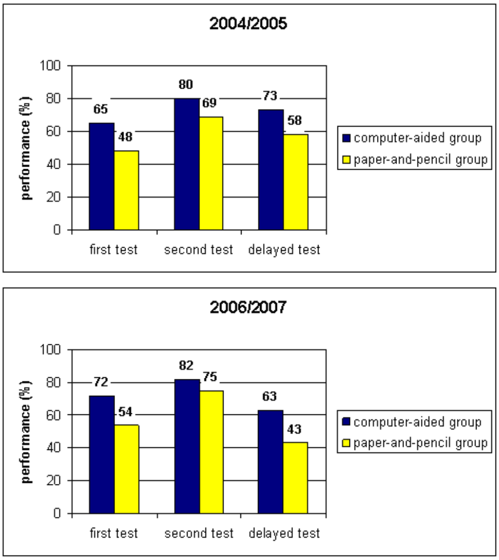


Figure 6: Results of two tests and delayed test

paper-and-pencil group. The difference between the performances in the first test was 17% in 2004, 18% in 2006. The difference between the performances in the second test was 11% in 2004, 7% in 2006.

As we compare the result of the delayed test with the tests we see that the performance of the paper-and-pencil group by all three tests of both years stayed under the performance of the computer group. In 2007 on the delayed test both groups performed worse than in 2005. The reason for this can be that the weekly number of the lecture decreased from two to one and also the fact that in 2007 they wrote the delayed test more than a month after ending the education comparing to 2005.

It cleared out from the answers to the questionnaire, and also during the conversations with the students that they liked that they could work with computer, they found constructions easier in this way. In average they visited the webpage from home used at practice once a week.

Based on the tests we can say that we can reach quality improving with using DGS. Organizing the education by computer takes much more time of the teacher, the effective usage of DGS requires continuous developing work, but the results of the tests show that the invested work returns.

In the computer group it was more typical that the students helped each other, corrected their mistakes. Experimentation was more typical for them as well, as the faulty elements could be hidden without any sign with a mouse click. The members of the paper-and-pencil group waited for the teacher's help, instruction when they stuck in their work. So the computer inspired the students for separateness. We found that the testing phase at the traditional, paper-and-pencil group was often missing.

As an effect of introducing the worksheets made by DGS into education the motivation level of learning increased, the worksheets are helpful for large percentage of the students.

## 5. Summary

The aim of our educational research was to introduce the worksheets made by DGS into the education of the Descriptive geometry of the mechanical engineer students. To reach this aim, after surveying of the literature of the spatial ability, the computer-aided education, the DGS and Mathematics Didactics, we executed a controlgrouped developing research for trying out the new educational method. At the creating and trying out of the curriculum made up of the DGS generated worksheets that include the material of the practises we took into consideration the offers of the literature and we modified and corrected the curriculumsystem according to the experiences of the first school year.

We may assert on the basis of these results that use of the computer and the use of interactive worksheets provided by DGS increases success and helps to create a proper conceptual structure. The computer-aided seminar helps the effectiveness of teaching, with the help of the interactive worksheets we can improve the student's problem-solving abilities and improvement in the field of creativity was observed among the students. On the seminar we could more easily trace the thoughts of the students [12].

Direction by the teacher is very importanteven in case of using DGS. If the software is simply made available, the program might become an obstacle to the transition from empirical to theoretical thinking, as it allows the validating of a proposition without the need to use a theory [1].

According to the current experiments, task of the future is to improve, develop and correct worksheets and with the help of all of these sheets we could make the tasks more efficient in the future.

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